
PERCENTS

The Dow (Dow Jones Industrial Average) rose **250 points**. The S&P 500 (Standard & Poor) rose **75 points**. Did the Dow gain more than the S&P? Don't be so sure!



A cancer treatment increases the chance of a heart attack by 90%. Is the cancer treatment worth the risk?

❑ UNDERSTANDING PERCENTS

“**Per**” means “divided by” (like *miles per hour* means miles **divided** by hours). “**Cent**” refers to “hundred” (like 100 years in a century, or 100 cents in a dollar). Therefore,

PERCENT means “**divided by 100**”

For any quantity X in the whole universe,

$$X\% = \frac{X}{100}$$

Since percent means “out of **100**” or “divided by **100**,” we can see what percents mean from the following examples:

$$1. \quad 17\% = \frac{17}{100}$$

$$2. \quad 9\% = \frac{9}{100}$$

$$3. \quad 32\% = \frac{32}{100} = \frac{8}{25}$$

$$4. \quad 6\% = \frac{6}{100} = \frac{3}{50}$$

$$5. \quad 125\% = \frac{125}{100} = 1\frac{25}{100} = 1\frac{1}{4}$$

$$6. \quad 800\% = \frac{800}{100} = 8$$

❑ **CONVERTING AMONG FRACTIONS, DECIMALS, AND PERCENTS**

Johnny sank 23 out of 32 free-throw attempts; Janie sank 29 out of 40 free-throw attempts. Who did better? Hard to tell — Janie made more baskets, but needed more attempts to do it. This section will show us how to determine who really did better. First, a couple of warnings:



1. There is to be **NO** rounding in this section; carry all divisions out until the remainder is 0.
2. All fractions must be reduced to lowest terms.

And notice that **100%** is just a fancy name for the number **1**. That is,

$$100\% = 1$$

$$\text{Proof: } 100\% = \frac{100}{100} = 1 \quad \checkmark$$

Decimal → Percent

Method: 1) Move the decimal point two places to the right.

2) Attach a percent sign.

Example: **Convert 0.007 to a percent.**

1) Move the decimal point: **0.7**

2) Attach a percent sign:

0.7%

Example: **Convert 23.6 to a percent.**

Moving the decimal point and attaching a percent sign gives the final answer of

2,360%

Percent → Decimal

Method: 1) Move the decimal point two places to the left.

2) Remove the percent sign.

Example: **Convert 3.5% to a decimal.**

1) Move the decimal point: **0.035%**

2) Remove the percent sign:

0.035

Percent → Fraction

This procedure was covered in the Section Understanding Percents.

Fraction → Percent

Let's solve the basketball problem with Johnny and Janie. We begin by expressing each player's stats as fractions.

$$\text{Johnny: } \frac{23}{32} \qquad \text{Janie: } \frac{29}{40}$$

To convert Johnny's score to a percent, first convert the fraction to a decimal, and then convert that decimal to a percent:

$$\frac{23}{32} = 0.71875 = \mathbf{71.875\%}$$

As for Janie's free-throw percentage:

$$\frac{29}{40} = 0.725 = \mathbf{72.5\%}$$

So we can conclude that Janie truly had the better stats.

Homework

1. Convert 56% to a reduced fraction.
2. Convert 5.9% to a decimal.
3. Convert .0045 to a percent.
4. Convert $\frac{4}{5}$ to a percent.
5. Convert 467% to a decimal.
6. Convert 3.89 to a percent.
7. Convert $\frac{7}{8}$ to a decimal.
8. Convert $\frac{33}{66}$ to a percent.

9. Convert 0.0077 to a percent.
10. Convert 0.025 to a reduced fraction.

❑ **PERCENT INCREASE AND DECREASE**

The price just went up \$20! Is that a big deal? Let's see.

Suppose the \$60,000 Lexus you ordered went up **\$20**; would you cancel your order? Of course not; \$20 out of \$60,000 is trivial, a drop in the bucket relative to the original price.

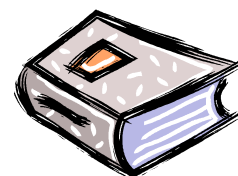


But what if the monthly \$25 fee for streaming Netflix increased by **\$20** to a total of **\$45**? Is that a big deal? Absolutely, and that's because \$20 out of \$25 is a huge increase, *relative to the original price*. Indeed, there's a really good chance that you might just cancel Netflix and get Disney+ instead.

How can the same \$20 mean so little in one case and so much in another? Because we have to ask ourselves, “\$20 compared to what?” Since a \$20 increase alone is meaningless, we must calculate the \$20 increase in relation to the starting price. The best way to do this is to calculate the **percent increase**, and it's found by dividing the increase (the \$20) by the original price (the \$60,000 car or the \$25 Netflix).

By the way, the Lexus increased in price by **0.03%**; the Netflix price increased by **80%**. The same \$20 increase — drastically different percent increases!

EXAMPLE 1: A \$12 book has just risen to \$15. Find the **percent increase** in the price of the book.

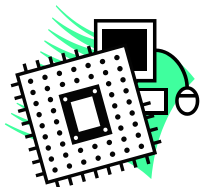


Solution: We're going to compute the increase in the price of the book, and then compare that increase to the original price:

$$\% \text{ increase} = \frac{\text{increase}}{\text{original}} = \frac{\$3}{\$12} = 0.25 = \mathbf{25\%}$$

Notice that the increase was found by subtracting the old price from the new price.

EXAMPLE 2: The price of an Intel i5 CPU chip has fallen from \$297 to \$103. Find the percent decrease.



Solution: We use the same logic as before. We compare the change in price with the original price. The price change is found by subtracting \$103 from \$297, giving \$194. Since the original price is \$297, we're ready to calculate the percent decrease:

$$\% \text{ decrease} = \frac{\text{decrease}}{\text{original}} = \frac{\$194}{\$297} \approx 0.653 \approx \mathbf{65\%}$$

EXAMPLE 3: Show that a percent increase can exceed 100%, but a percent decrease can never exceed 100%.

Solution: Consider that the price of gasoline increased from \$0.50/gal in 1970 to \$3.15/gal in 2020. We calculate the percent increase in the price of gasoline over this 50-year span like this:



$$\% \text{ increase} = \frac{\text{increase}}{\text{original}} = \frac{\$2.65}{\$0.50} = 5.3 = \mathbf{530\%}$$

In fact, if something doubles, that's a 100% increase (you'll verify this fact in the homework), so any increase that's more than doubling will exceed a 100% increase. Therefore, there is no limit

on a percent increase; it could be a 6 billion percent increase and still be a valid calculation.

As for the highest possible percent decrease, consider this: A 100% decrease would be the removal of all of it. Since there's nothing left to remove, a decrease can never exceed 100%. For example, suppose that 200 frogs decreased in population by 100%. That means that every single one of them croaked. How can the frog population decrease any more than that? It can't, so a percent decrease is at most 100%.



Summary: In all three examples, we calculated the percent change (whether an increase or a decrease) by creating a fraction with the difference of the quantities on the top and the original quantity on the bottom. When that fraction was converted to a percent, we were done.

$$\text{Percent Change} = \frac{\text{Difference}}{\text{Original}}$$

Homework

Round your answers to the nearest whole percent:

11. Marie's wage increased from \$8/hr to \$12/hr. Find Marie's percent increase in wage.
12. The library increased its collection from 2500 books to 4200 books. Find the percent increase in the number of books.
13. Jim's weight went from 150 lb to 227 lb. Calculate the percent increase in weight.
14. The number of TB cases increased from 32 to 97. Calculate the percent increase in the number of TB cases.
15. The number of tumors increased from 49 to 88. Calculate the percent increase in the number of tumors.
16. The number of K rats increased from 12 to 42. Calculate the percent increase in the number of K rats.
17. The price of the Geo went from \$14,000 to \$19,000. The price of the Lexus went from \$52,000 to \$72,000. In terms of percent increase, which car increased in price more?
18. The price of top sirloin has doubled. Find the percent increase.
[Hint: Make up your own numbers that represent a doubling of price.]
19. The number of swimmers has tripled. Find the percent increase.
20. The price of gasoline has quadrupled. Find the percent increase.
21. Thanks to Phoebe's "Smelly Cat" song, the number of cats has quintupled (yuk!). Find the percent increase.
22. a. Explain how it is possible that something could increase more than 100%.
b. Explain why it is impossible for something to decrease more than 100%.



23. The number of hospitals decreased from 45 to 34. Calculate the percent decrease in the number of hospitals.
24. The number of K rats decreased from 22 to 10. Calculate the percent decrease in the number of K rats.
25. The number of prisons decreased from 52 to 36. Calculate the percent decrease in the number of prisons.
26. The price has dropped from \$200 to \$6. Find the percent decrease.
27. The number of dogs has fallen from 250 to 195. Find the percent decrease.
28. The number of heart attacks went from 760 before the drug was available to 490 after the drug was available. Calculate the percent decrease in heart attacks.
29. The number of dogs increased from 55 to 93. Calculate the percent increase in the number of dogs.
30. If the number of prisons increased by a factor of 6, make up your own numbers to find the percent increase. [An increase “by a factor of 6” means that the number of prisons is now six times what it was before.]
31. If the number of houses increased by a factor of 7, find the percent increase.
32. If Jesse’s salary increased by a factor of 9, find the percent increase.
33. If the population increased by a factor of 12, find the percent increase.
34. The number of motels decreased from 55 to 14. Calculate the percent decrease in the number of motels.
35. The number of military personnel decreased from 10,750 to 8,900. Calculate the percent decrease in the number of military personnel.
36. The number of classrooms decreased from 152 to 132. Calculate the percent decrease in the number of classrooms.
37. While the school-aged population went from 6,750 to 9,023, the number of teachers went from 225 to 190. Calculate the percent increase in students and the percent decrease in teachers.



38. The tuition might double next semester. What percent increase would this be?
39. What percent increase will there be if the tuition triples?

Solutions

1. $\frac{14}{25}$ **2.** 0.059 **3.** 0.45% **4.** 80% **5.** 4.67

6. 389% **7.** 0.875 **8.** 50% **9.** 0.77% **10.** $\frac{1}{40}$

11. 50% **12.** 68% **13.** 51% **14.** 203%

15. 80% **16.** 250%

17. The Lexus increased more (by 38%) than the Geo (by 36%).

18. Let's make up our own numbers and see what happens. Suppose top sirloin was \$2.25/lb and then the price doubled to \$4.50/lb. Calculate the percent increase:

$$\text{percent increase} = \frac{\text{difference}}{\text{original}} = \frac{4.50 - 2.25}{2.25} = \frac{2.25}{2.25} = 1 = \mathbf{100\%}$$

Now make up your own numbers to see if the same result is obtained.

19. 200% **20.** 300% **21.** 400%

22. a. When something doubles, it increases by 100%. So if something more than doubles, it would increase by more than 100%.
b. Once it's decreased 100%, all of it is gone — it can't decrease any further.
23. 24% 24. 55% 25. 31% 26. 97%
27. 22% 28. 36% 29. 69% 30. 500%
31. 600% 32. 800% 33. 1,100% 34. 75%
35. 17% 36. 13%
37. Students increased by 34%. Teachers decreased by 16%.
38. 100% 39. 200%

**“It is common sense to take a
method and try it.
If it fails, admit it frankly and
try another.
But above all, try something.”**

Franklin D. Roosevelt